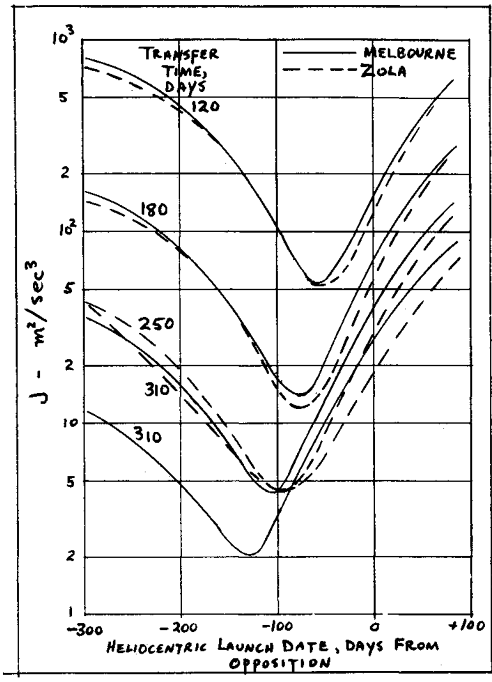

Fig. 1 Characteristic length (Zola) results compared with Melbourne's data for Earth-Mars transfers.

transfers at each transfer time and polar travel angle. The equations used for these calculations are

$$L = T \Delta V_{HT}/2 \tag{1}$$

$$J = 12L^2/T^3 \tag{2}$$

where L is the characteristic length and T is the actual transfer time. A more detailed description of this method is given in Refs. 2 and 3. The corrected figure shows that the error in the length method does not grow rapidly until the transfer time exceeds the Hohmann time (~260 days). This is because the single-conic, high-thrust transfers used in this data do not give minimum ΔV_{HT} beyond the Hohmann time.

The characteristic length correlation is a method of predicting the performance of one mode of rocket operation from known performance of some other mode. One use, if desired, is predicting variable thrust data from high-thrust data, as in Fig. 1 of this comment, but its primary purpose is to help analyze the performance of low-thrust spacecraft with constant thrust and specific impulse. Reference data for the correlation can be precalculated solutions of the trajectory problem using constant-thrust, variable-thrust, or high-thrust operation.

The characteristic length method has many accuracy limitations that are discussed in Ref. 3. Also noted in Ref. 3 is that low-thrust data at constant thrust and specific impulse are better predicted with variable-thrust data as a reference, not the single-conic, high-thrust data used in Fig. 1. Furthermore, as noted by Ragsac,[1] recent computing methods make variable-thrust solutions obtainable in a matter of seconds, if not already available in published form.

Although the accuracy of the length correlation may vary from mission to mission, it is most accurate for Mars and Venus orbiter trajectories. Also, MacKay[5] et al. and Brown[6] have successfully used the method for constant low-thrust mission studies which include combined high- and low-thrust operation. The effect of combined thrust operation is estimated by assuming that the high-thrust maneuvers at either end of the path reduce the required ΔV_{HT}. The reduced ΔV_{HT} means lower L and leads to lower propellant and power for the low-thrust spacecraft. MacKay[5] et al. have found this method to be accurate to within 5% of exact calculations for required initial orbiting mass for Earth-Mars round trip missions.

References

[1] Ragsac, R. V., "Study of electric propulsion for manned Mars missions," J. Spacecraft Rockets **4**, 462–468 (1967).

[2] Zola, C. L., "Trajectory methods in mission analysis for low-thrust vehicles," AIAA Paper 64-51 (1964).

[3] Zola, C. L., "A method of approximating propellant requirements of low-thrust trajectories," NASA TN D-3400 (1966).

[4] Melbourne, W. G. and Sauer, C. G., "Optimum Earth-to-Mars round trip trajectories utilizing low-thrust power-limited propulsion systems," Jet Propulsion Lab. TR 32-376 (March 1963).

[5] MacKay, J. S. et al., "Manned Mars landing missions using electric propulsion," NASA TN D-3194 (1966).

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Reply by Author to C. L. Zola

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In the comment by Zola, it is stated that the comparison in Fig. 1 of Ref. 1 uses the parameter $J = \frac{1}{2}\int a^2dt$. However, the parameter employed is $J = \int a^2dt$; this fact is noted[1] where Fig. 1 is first cited. The curve for the 250-day transfer time (variable thrust) was carefully checked using the J from Eqs. (1) and (2) of Zola's comment and the hyperbolic excess speeds listed in the NASA Planetary Flight Handbook, SP-35. As required by Zola's method,[2] the reference-mode solution must correspond to the same trajectory problem in the new mode. Since comparisons are being made for heliocentric transfer trajectories between planets moving in mutually inclined, elliptic orbits (i.e., three-dimensional), the corresponding hyperbolic excess speeds from SP-35 were used. The results are shown in Fig. 1. The use of more data points uncovers a discontinuity not previously noted. Because of the three-dimensional effects, the hyperbolic excess speed values at certain planetary configurations (launch dates) rise steeply and then descend, forming a "ridge."[4] Consequently the effect on J should be quite similar. Other than this ridge, the curve remains essentially the same as originally presented in Ref. 1.

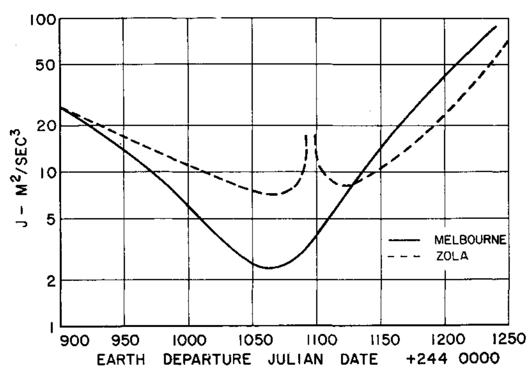

Fig. 1 Characteristic length (Zola) results compared to Melbourne's data for 250-day Earth-Mars trip.

Received July 6, 1967; revision received July 19, 1967.

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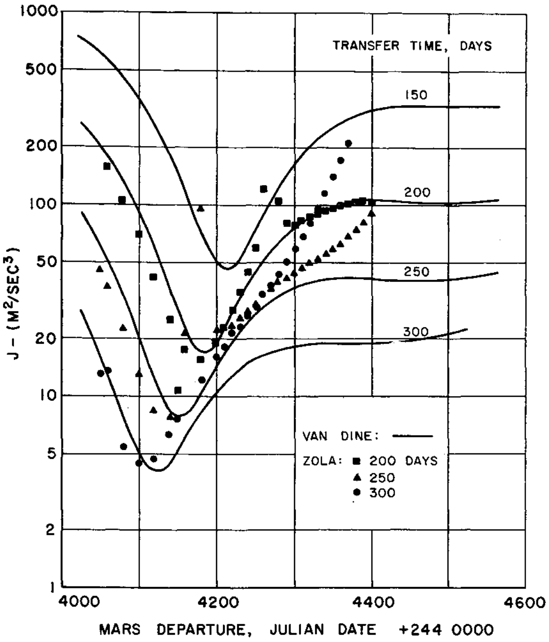

Fig. 2 Zola's results (data points) compared to Van Dine's for 1980 Mars to Earth trips.

This ridge effect was encountered in other trajectories when the same comparisons were made. Figure 2 compares the length method against Van Dine's results for Mars to Earth trips in 1980. At a trip time of 200 days, and neglecting the ridge effect, Zola's data agree rather well with Van Dine's results. The agreement, however, breaks down at the 250-day trip time, especially for trips departing from Mars after the minimum-J departure date.

The discrepancy between the results in Zola's comment and that presented here is the fact that Zola's data are based on circular, coplanar, planetary orbits. Under these conditions Zola's data agree with Melbourne's corresponding results up to about the Hohmann transfer time. It is of interest to note that by comparing three-dimensional J's against those from circular, coplanar assumptions, the values differ by about a factor of 2. This gives rise to the question of the factor $\frac{1}{2}$ in the definitions for J.

Furthermore, these differences point up the problem of using circular, coplanar-based impulsive-thrust data to estimate low-thrust trajectory requirements which, consequently, do not necessarily represent any particular opposition year. This problem is indicated in Fig. 3 wherein the circular, coplanar curve is bracketed by the three-dimensional data from

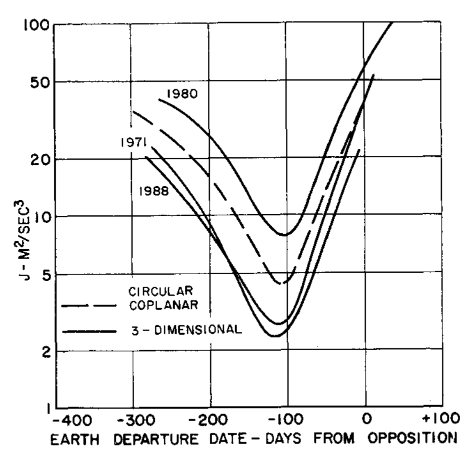

Fig. 3 Comparison of J requirements resulting from circular, coplanar orbits and three-dimensional planetary orbits; 250-day Earth-Mars trips.

the 1971, 1980, and 1988 oppositions. For analyses in which the opposition year effects are not important, the circular, coplanar data are quite useful. Obviously, if the year of launch is important, the reference-mode data should reflect this. Between 1971 and 1980, the J requirements can change by about a factor of 3.

As Zola indicates, the primary purpose of the length method is to aid in the evaluation of constant-specific impulse, low-thrust systems. For these evaluations, high-thrust data, whether from circular, coplanar trajectories or from SP-35, are not desirable reference-mode solutions for the reasons shown here and in Ref. 3. This is particularly true for round-trip missions if one leg has a launch date and a trip time much different from that permitted by the accuracy of the characteristic-length technique (e.g., see Fig. 2).

If quantities of variable-thrust data are available as reference-mode solutions, Zola's technique is quite helpful in estimating constant-thrust performance (Mars and Venus missions). Under these circumstances, it is a most efficient means of generating approximate trajectory data and of performing mission studies.

References

[1] Ragsac, R. V., "Study of electric propulsion for manned Mars missions," J. Spacecraft Rockets 4, 462–468 (1967).

[2] Zola, C. L., "Trajectory methods in mission analysis for low-thrust vehicles," AIAA Paper 64-51 (1964).

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Comment on "A Method for Calculating Parachute Opening Forces for General Deployment Conditions"

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FIVE comments are offered here on Jamison's recent article[1] (equation numbers and nomenclature the same as Jamison's):

1) Jamison does not appear to be acquainted with Pounder's work,[2] which remains the most thorough and rational treatment yet available on the problems of parachute inflation and opening shock.

2) Jamison assumes that the diameter vs time relationship is

$$\beta = a + bT + cT^2 + dT^3 \tag{13}$$

with boundary conditions $\beta(0) = \beta_0$, $\beta(1) = \beta_f$, $\dot{\beta}(0) =$

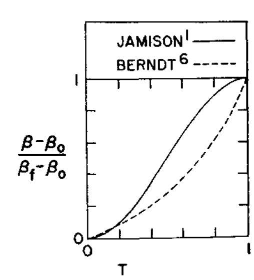

Fig. 1 Comparison of Jamison's opening function with Berndt's experimental data.

Received May 18, 1967.
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